SPECTRAL CONSEQUENCES OF DEVIATION FROM SPHERICAL COMPOSITION SYMMETRY IN TYPE IA SUPERNOVAE

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ABSTRACT

We investigate the prospects for constraining the maximum scale of clumping in composition that is consistent with observed Type Ia supernova flux spectra. Synthetic spectra generated without purely spherical composition symmetry indicate that gross asymmetries make prominent changes to absorption features. Motivated by this, we consider the case of a single unblended line forming in an atmosphere with perturbations of different scales and spatial distributions. Perturbations of about 1% of the area of the photodisk simply weaken the absorption feature by the same amount independent of the line of sight. Conversely, perturbations of about 10% of the area of the photodisk introduce variation in the absorption depth which does depend on the line of sight. Thus, 1% photodisk area perturbations may be consistent with observed profile homogeneity but 10% photodisk area perturbations can not. Based on this, we suggest that the absence of significant variation in the depths of Si II λ 6355 absorption features in normal Type Ia spectra near maximum light indicates that any composition perturbations in these events are quite small. This also constrains future three-dimensional explosion models to produce ejecta profiles with only small scale inhomogeneities.

Subject headings: hydrodynamics — radiative transfer — supernovae: general

1. INTRODUCTION

Three-dimensional calculations of white dwarf deflagrations suggest significant deviation from spherical composition symmetry in the resulting Type Ia supernova (SN) envelopes (Khokhlov 2000; Hillebrandt et al. 2000). Yet the well-documented spectral homogeneity of normal events tightly constrains the model output spectra. This inspires us to consider a very general question relevant to three-dimensional calculations of white dwarf explosions: How much and what kinds of deviation from spherical composition symmetry might still produce morphologically plausible and homogeneous spectra from all lines of sight?

The present Letter addresses this question using some simple arguments. In §2, using a modified version of the direct analysis code SYNOW, we generate some sample spectra to demonstrate artifacts caused by deviation from spherical composition symmetry. Motivated by these results, we pursue in §3 a possible means of constraining deviation from spherical composition symmetry by considering the formation of a single unblended line in an envelope with a simple composition perturbation. The small scatter in the depths of Si II absorption features in normal Type Ia spectra near maximum light may indicate that any composition perturbations present exist only on small scales.

2. CLUMPY SN SPECTRA

The fast, parameterized, SN spectrum synthesis code SYNOW has been used for "direct" analysis (Fisher et al. 1997) of several SNe of varying types [e.g., Type Ia SN 1994D (Hatano et al. 1999), Type Ib SN 1999dn (Deng et al. 2000), Type Ic SN 1994I (Millard et al. 1999), Type II SN 1999em (Baron et al. 2000)]. The goal of direct analysis is to establish line identifications and intervals of ejection velocity within which the presence of lines of various ions is detected, without adopting any particular hydrodynamical model. Composition and velocity constraints obtained from SYNOW then can provide guidance to those who compute hydrodynamical explosion models and to those who carry out computationally intensive NLTE spectrum modeling. A complete description of the workings of SYNOW can be found in Fisher (2000). Here we present only the background necessary to understand the modifications made to allow SYNOW to produce spectra from model envelopes without spherical composition symmetry.

2.1. *SYNOW and ClumpySYN*

In its simplest form, SYNOW uses spherical composition symmetry and the assumption of resonance scattering for line formation. Line formation takes place in an envelope surrounding a sharp photosphere, a source of continuum radiation. SYNOW treats line formation in the Sobolev approximation, which is a good one for analysis purposes. The profile of a line is determined by the adopted radial distribution of the line optical depth and the line source function.

The geometric algorithm that SYNOW uses to calculate the line source function at a given point in the envelope requires some explanation. Consider an atmosphere like that depicted in Figure 1. The shaded region represents the sharp photosphere, and we draw an imaginary boundary at v_{max} where the optical depth in all lines we are

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considering drops to a negligible amount. Our goal is to calculate the source function in some line with wavelength λ at the point P.

Adopting the simple explosion velocity law $v \propto r$ implies that surfaces of constant velocity with respect to the point P in the atmosphere are spheres centered on that point. In an envelope with spherical composition symmetry, points physically relevant for computing the line source function at P are those on such common point velocity spheres which are within v_{max} of the explosion center and are not occulted by the photosphere. Two such surfaces are labelled A and B in the figure. The cone C denotes the boundary of an occultation region produced by the photosphere.

Now consider two other lines with wavelengths λ_A and λ_B such that $\lambda = \lambda_A(1 + \Delta v_A/c) = \lambda_B(1 + \Delta v_B/c)$ where Δv_A and Δv_B are the velocity radii of spheres A and B respectively. Photons that redshift with respect to the matter into resonance with the line λ_B along the surface B scatter isotropically. Some of these are redirected towards P. These photons redshift further until they come into resonance with the line λ_A at surface A. Here they can be scattered away from P or proceed unhindered to redshift into resonance with the line λ at P. Similarly, photons emitted from the photosphere that redshift into resonance with the line λ_A on the sphere A can be scattered towards or away from P. By computing the intensity arriving at P from many surfaces and the photosphere, the source function in the line λ at P can be built up. Clearly, computing the source function as a function of radius is the most intensive part of a SYNOW calculation as the program considers a long list of lines. Even so, an entire spectrum covering many lines can be produced in minutes.

One special case that occurs when calculating the source function is when no shorter wavelength lines are close enough to λ in wavelength to be scattered from a corresponding surface within the sphere v_{max}. In such a case, the line is *unblended* and the source function consists only of the photospheric intensity times the geometric dilution factor of the photosphere.

When spherical composition symmetry is broken, a particular scheme must be chosen to parameterize the line optical depths on a three-dimensional grid instead of along a radius. We call the code we use in this case "ClumpySYN," and choose to group together regions in the atmosphere in spherical "clumps." We can confine species to the clumps by setting the corresponding line optical depths to zero everywhere else. Conversely, we can exclude species from clumps by setting corresponding line optical depths to zero within clumps while assigning nonzero values elsewhere. We still adopt a particular radial profile ($\tau \propto \exp(-v/v_e)$) to determine the value of the line optical depth at a point if it is allowed to be nonzero there.

Unfortunately, distributing line optical depth in a way that breaks spherical symmetry introduces a new wrinkle. Figure 2 illustrates that when optical depth is distributed in an arbitrary fashion, different regions of the surfaces A and B contribute to the intensity at P in an angular dependent way. The above geometric algorithm for determining the source function everywhere can either be generalized to handle such arbitrary optical depth distributions (instead of a simple function of radius only) or we can ignore

line blending. When we compare two spectra generated with the same spherically symmetric composition but with and without line blending (or multiple scattering), we find that the differences between them are minor. Hence, in ClumpySYN we choose to neglect multiple scattering when calculating the source function. Now the source function in each line simply equals the photospheric intensity times the geometric dilution factor of the photosphere.

A new geometric algorithm could be developed to compute the source function including multiple scattering, but since SYNOW makes several assumptions already (i.e., pure presonance scattering source function, a sharp photosphere) such an improvement seems only marginally profitable. This is especially clear in light of the similarity of spherically symmetric synthetic spectra with and without multiple scattering.

2.2. *Sample ClumpySYN Spectra*

An example of a clump configuration is shown in Figure 3. Here, we place an upper velocity boundary at 25000 km s^{-1}. We allow the clumps to overlap and have radii in velocity space between 5000 and 6000 km s^{-1}. The fraction of the volume taken up by clumps in the envelope (between the photosphere at 11000 km s^{-1} and the upper boundary) in Figure 3 is about 66%. Several such models were generated with different volume filling factors, and model output spectra were computed from several lines of sight with ClumpySYN.

Motivated by Figure 12 of Khokhlov (2000), we partition the clumps into two parts. In the inner part of the clumps, we place Fe II ions. In the outer part, we place intermediate mass ions (Si II, Ca II, S II). Outside the clumps, we place O I. The choice of these particular ions is motivated by a maximum light fit of SN 1994D (Hatano et al. 1999).

Figure 4 presents synthetic spectra from models with different envelope filling factors. Each graph displays four spectra, one each of four different lines of sight spaced 90 degrees apart about the equator. We note that higher volume filling factors cover the photosphere more effectively and the spectra are quite similar along all lines of sight. At lower filling factors, perspective-dependent spectral diversity begins to creep in, particularly in absorption features.

3. THE THRESHOLD CLUMPING SCALE

Previous studies of SN spectra from geometries departing from spherical symmetry are polarization investigations (e.g., Howell et al. 2001; Wang et al. 1997) and some axisymmetric configuration line profiles (Jeffrey & Branch 1990). These consider ellipsoidal deformations of the envelope. Here we are concerned with less global perturbations in composition.

To illustrate clearly the effects of composition clumping on spectra, we consider the formation of a single unblended line in an expanding SN envelope. The standard assumptions of SYNOW we also apply here: homologous expansion and resonance scattering. In this simple exercise, we characterize the degree of clumping with two parameters. The first parameter governs the cross sectional area of an individual clump relative to the area of the photodisk (the photosphere's cross section along a line of sight). The second controls the spatial deployment of the clumps, their sparsity or density. Within clumps, line

optical depth is defined relative to its value at the photosphere ($\tau \propto \tau_{phot} \exp(-v/v_e)$).

With the line source function known everywhere and τ assigned to clumps, computing the line profile is simple. We integrate over each surface of common velocity relative to an observer at infinity to obtain the flux as a function of velocity (or observer frame wavelength). In homologous flow, these surfaces are planes perpendicular to the line of sight.

We define the "photodisk covering factor" for a plane of common velocity as the ratio of the area in clumps intersecting that plane which are contained in the projected photodisk area to the total photodisk area. In a spherically symmetric composition, τ is greatest at the photosphere so the deepest part of the absorption feature corresponds to the common velocity plane tangent to the photosphere (unless the line is very strong). Clearly, decreasing the covering factor in this plane weakens the absorption minimum relative to the "totally covered photodisk" minimum. One could compensate for this weakening by increasing τ_{phot}, but there is a limit to how effective this can be; even as τ_{phot} is increased to infinity, the minimum flux will saturate because the uncovered portion of the photodisk shines through. This implies that for a given observed absorption depth fit by some choice of τ_{phot} assuming total photodisk coverage, there is some minimum covering factor determined by $\tau_{phot} \to \infty$ that could fit the same line.

If the average clump is much smaller in cross section than the photodisk area, the covering factor will tend to be the same value from all lines of sight, inducing only limited perspective-dependent absorption depth diversity. On the other hand, if the average clump has a large cross section comparable to the photodisk area, observers with different lines of sight will measure different line profiles. The covering factor as the line of sight is shifted will vary enormously.

To illustrate these facts, τ is assigned to a three dimensional Cartesian grid using the following "white noise" prescription. The grid is partitioned into regular cubical cells with s grid points along each edge. The list of cubical cells is then traversed and at each cell a uniform random deviate is chosen. If the deviate is less than a prescribed value f_c, τ is assigned to each grid point contained within the cell according to the exponential τ profile, otherwise τ is set to zero at all points in the cell. In the end, for N cells, about $f_c N$ will be cubical clumps and the remainder will be empty. We choose to compute line profiles for six observers situated at infinity such that the lines of sight are perpendicular to cell faces so the value of f_c will roughly equal the fraction of grid points within clumps on each common velocity plane. The cubical shape of the clumps causes the line profiles to be jagged, but other choices will just smooth out the line profiles and not change the general result, since the photodisk covering factor is the important quantity.

In Figures 5a and 5c, line profiles for two values of s (8 and 32 grid points respectively) are shown. In each figure, five values of f_c are shown, and six different viewpoints of each realization are superimposed. The photodisk radius is 50 grid points and the optical depth at the photosphere is set to 7.5. The optical depth falls off with radius exponentially, with an e-folding length of 15 grid points. In Figure 5a, the ratio of each clump cross section to the photodisk area is small, about 1/120. Hence, for all values of f_c, the line profiles are quite similar from all lines of sight. However, in Figure 5c, each clump cross section is 1/8 the area of the photodisk for the six observers, so changing perspective radically alters the actual fraction of the photodisk covered. This introduces perspective-dependent line profile diversity.

Figures 5b and 5d are plots of absorption feature depth versus f_c. For $s = 8$ (5b), the depths measured by each observer are close together while for $s = 32$ (5d), the depths are scattered markedly. In both figures, a vertical line indicates the lowest f_c value where τ_{phot} could be increased to match a $f_c = 1$ observation. A collection of spectra which exhibit significant absorption depths consistent across the observed sample indicate that any clumps present must be small and densely deployed, i.e., a consistent, deep line implies that the photodisk is nearly covered by optical depth from all lines of sight.

In normal Type Ia SNe, one possible spectrum feature that might help constrain the actual amount of clumping is the distinctive Si II doublet ($\lambda\lambda$ 6347, 6371 Å) near maximum light. We studied a sample of good quality maximum light spectra obtained over the last two decades and found that measured Si II depth does not vary appreciably from event to event. A few examples in the wavelength range of interest are shown in Figure 6. The average value of the absorption depth is about 0.67 relative to the continuum, with a standard deviation of 0.06. If we interpret these observations as instances of the same class of event observed from different lines of sight, the lack of substantial scatter implies that the composition clump scale of Si in normal Type Ia SNe is similar to that used to create line profiles in Figure 5a. Any clumps of Si present must be smaller than the detection threshold for clumping, which we have illustrated to be set by clumps much smaller than 1/10 the area of the photodisk.

4. CONCLUSION

By effecting minor modifications to the direct analysis code SYNOW, we have produced ClumpySYN, a code that can generate spectra from compositions without spherical symmetry. Based on a simple parameterization, we suggest that the important factor for spectra from events lacking a spherical composition is the fraction of the photosphere covered by clumps. If the clumps are large compared to the size of the photosphere, inhomogeneity in unblended line absorption becomes manifest. Below a threshold scale, these clumps only weaken absorption along different lines of sight by the same amount. Absorption strength accounted for by spherically symmetric compositions can be recovered in clumpy models by increasing optical depth, but only to an extent.

In addition, we suggest that the robustness of absorption depth measurements in the Si II feature in normal Type Ia SNe implies that any perturbations in composition away from spherical symmetry are smaller than the threshold scale. From this, one might conclude that if a general characteristic of deflagration models is the formation of large clumps of Si and Fe, then normal Type Ia SNe are not the result of deflagrations. More conservatively, we suggest that explosion models must avoid generating large-

scale bubbles or clumps in composition to recover normal Type Ia SN spectral homogeneity.

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Fɪɢ. 1.— SYNOW computes the line source function at P by considering photons scattered there from cylindrically symmetric common point velocity surfaces A and B. The horizontal dashed line is the axis of symmetry for the common point velocity surfaces as observed from P.

Fɪɢ. 2.— In cases where optical depth is not distributed in a spherically symmetric way, photons scatter toward P in an angular dependent way.

Fɪɢ. 3.— A sample clumpy model with 66% of the envelope filled.

Fɪɢ. 4.— Spectra resulting from ClumpySYN calculations with different envelope filling factors (labelled in each panel). A synthetic spectrum similar to a maximum light fit to SN 1994D (Hatano et al. 1999) is the dashed line.

Fɪɢ. 5.— (a) Six line profiles each for $f_c = (0.2, 0.4, 0.6, 0.8$ and $1.0)$ from top to bottom with small-area perturbations. The lines are offset by $C = (1.2, 0.9, 0.6, 0.3$ and $0)$ from top to bottom. (b) A corresponding plot of absorption depth as a function of f_c for the small perturbations. Even if the line optical depth is increased to infinity, only for $f_c \gtrsim 0.7$ can the $f_c = 1$ depth be recovered. (c) Same as (a) for large-area perturbations. (d) The depth scatter compared to that for the small-area perturbations is much larger. Increasing optical depth will on average recover the absorption depth for $f_c = 1$, but too much scatter in the absorption depth will be incurred.

Fɪɢ. 6.— Sample Type Ia Si II features at maximum light (about 20 d after explosion) for SN 1989B (Wells et al. 1994), SN 1990N (Mazzali et al. 1993), SN 1994D (Patat et al. 1996) and SN 1998bu (Hernandez et al. 2000).

A

B

B

C

P

v_{max}

ϕ